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EIGEN NOISE IN WIDE-BAND ELECTROMETRIC AMPLIFIERS

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ABSTRACT

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The eigen noise of wide-band electrometric amplifiers is computed. The conditions for obtaining maximum sensitivity are determined. The effect of the parameters of the input circuit, of the tube and of the type of scheme on the value of the maximum sensitivity are considered.

Author

Introduction

In electrometric amplifiers with high sensitivity to current and a relatively large time constant of the input circuit-$\tau > 0.1$ sec (with a narrow-band width from 0 to 1-2 cps) the sensitivity is usually limited by the internal noise of the electrometric tube (refs. 1,2). Of the three components of noise in the input circuit - thermal noise of the resistance, the noise of grid current and the flicker-noise of the tube - the latter is most pronounced. As we shall show, the current of the thermal noise of the input high megohm resistance decreases with an increase in the value of the resistance; the resulting narrowing of the band-width may be compensated for by negative feedback (ref. 3). The noise due to the grid current may be decreased by selecting proper operating points for the electrometric tube which provides for a sufficiently low grid current (ref. 4). The flicker-noise can only be reduced by taking appropriate measures in the design and construction of the tube.

[*]Numbers given in the margin indicate the pagination in the original foreign text.

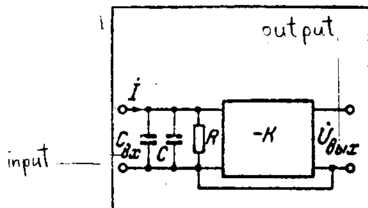
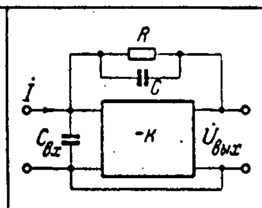
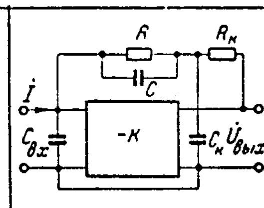
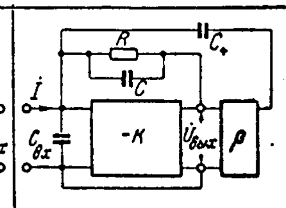
However, the proposition of the prevalence of flicker-noise in the total noise of an electrometric amplifier is valid only when we have a relatively narrow band pass. In wide-band electrometric amplifiers (with a band pass up to several hundred cycles) which are widely used in connection with the requirements of biophysics and automation, when modern electrometric tubes are utilized, there is a prevalence of thermal noise produced by the high resistance and the sensitivity is close to the theoretical limit.

Below we present the calculations of the internal noise in wide-band electrometric amplifiers with compensation for several typical values of the parameters of the input circuit and the constants of the flicker-noise tubes as well as the results of noise measurements conducted on bread-board models which confirm these calculations.

1. Calculation of the Internal Noise of the Input Circuit

In wide-band electrometric amplifiers (as well as in narrow-band electrometric amplifiers) it is desirable to increase the input high megohm resistance in order to increase the signal-to-noise ratio. In this case to increase the band-width it is insufficient to provide for the conventional negative feedback /95 through the high resistance. It becomes necessary to compensate for the frequency characteristics of the input circuit either by using additional positive feedback or by adding a compensating filter in the negative feedback loop. Both methods of compensation are described in detail, e.g., in ref. 5 ; in this article we shall present simplified equivalent schemes and equations for the input admittance which are used for computing the noise. For comparative purposes we also compute the noise in a scheme with feedback without compensation and in a scheme without negative feedback but with a high megohm resistance which has been decreased by a factor of $1 + K$. The input impedances for these four variations of the electrometric amplifier are shown in Table 1. As usual in the derivation of equations it is assumed that the gain without feedback (when the input is shortcircuited) K does not depend on frequency and that the output admittance of the amplifier is much greater than the transfer admittance of the feedback loop Y.

Table 1

Without Negative Feedback (oc : feedback)	With Negative Feedback without compensation	Compensated according to the method of Pelchowitch	With compensation and positive feedback
			
$Y' = Y = R^{-1} + j\omega C$ $\tau = R(C + C_{\theta x})$	$Y' = Y(1+K) = (1+K)/R + j\omega C(1+K)$ for $C(1+K) \gg C_{\theta x}$ $\tau = \frac{R}{1+K} C(1+K) = RC$	$Y' = Y(1+K\dot{\beta}_K) = (R')^{-1} + j\omega C'$ $\dot{\beta}_K = (1+j\omega\tau_K)^{-1}$; $\tau_K = R_K C_K$ For $K \gg 1$ u $\tau \approx \tau_K$ $\frac{1}{R'} \approx \frac{1+K}{R} \frac{1+\omega^2\tau\tau_K}{1+\omega^2\tau_K^2}$ $C' \approx C(1+K)\frac{1-\tau/\tau_K}{1+\omega^2\tau_K^2}$	$Y' = Y(1+K) + Y_+(1-K\dot{\beta})$ $Y_+ = j\omega C_+$ $(R')^{-1} = (1+K)/R$ for $K \gg 1$ $C' = C(1+K) + C_+(1-K\beta) \approx K(C - \beta C_+)$

Since in the considered schemes the feedback is produced by a linear transformation of the input admittance and since the different variations of the scheme differ only in the magnitude of the coefficients of transformation, all of the four schemes may be reduced to one single equivalent scheme without feedback in which the equivalent input admittance $Y_{oc} = Y_{in} + Y'$ consists of two parts--admittance Y_{in} (connected directly between the input terminals and not transformed by the feedback) and a transfer admittance Y' (transformed by feedback). This unified equivalent scheme is shown in figure 1. The four-pole network K is an ideal amplifier with transfer coefficients which are independent of frequency and with an input admittance equal to zero. The admittances of a real four-pole network are taken into account in the values of Y_{in} and Y.

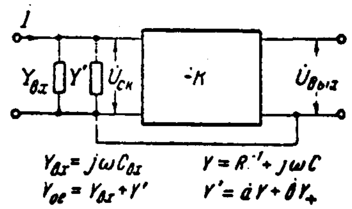


Figure 1. The generalized equivalent scheme for an amplifier with feedback.

The equivalent scheme of the amplifier, which shows how the basic sources of noise in the input circuit are connected, is shown on figure 2. The thermal noise of the high megohm resistance R is represented by a voltage generator whose mean square value is equal to:

$$\overline{E}_R^2 = 4kTR\Delta f,$$

where k = 1.37×10^{-23} joules/$^{\circ}$C is the Boltzmann constant, T $\approx 300^{\circ}$K. The shot noise of the grid current of the electrometric tube is represented by a current generator: /96

$$\overline{I}_{\text{Д}}^2 = 2eI_c\Delta f,$$

D

where e = 1.66×10^{-19} coulombs is the charge on the electron, I_c is the sum of the absolute positive and negative components of the grid current. The flicker-noise of the electrometric tube is represented by a voltage generator which feeds the cathode and is approximately described by the equation

$$\overline{E}_{\Phi}^2 = Af^{-\alpha}\Delta f,$$

f

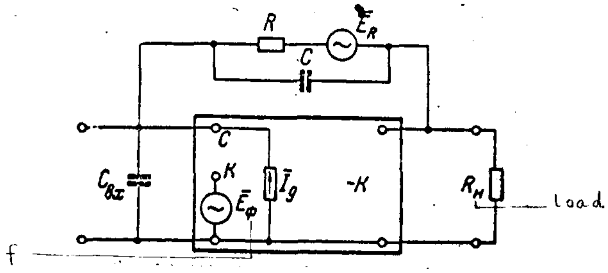


Figure 2. Basic sources of internal noise in an amplifier with feedback.

where A is a coefficient which depends on the type and operating state of the tube; α is an exponent which characterizes the distribution of the flicker-noise energy in the frequency spectrum. In the electrometric amplifier, of course, there are other sources of internal noise, e.g., the thermal noise of the plate load of the first tube; however, its effect is very small, particularly in modern electrometric pentodes, which provide for a stage gain of more than 10.

To simplify and unify the calculations it is rational to transform the voltage generators of noise into current generators. For the voltage produced by thermal noise this transformation follows from the well-known theorem concerning the equivalent generator:

$$\overline{I}_R^2 = \overline{E}_R^2/R^2 = 4\,kT\Delta f/R.$$

In transforming the voltage generator of flicker-noise into the equivalent current generator it is necessary to take into account feedback. The voltage of the flicker-noise between the grid and the cathode of the first tube $\dot{U}_{fgc}$ is obtained by subtracting from the e.m.f. $\dot{E}_f$ the feedback voltage $\dot{U}_{fgc}$ = $\dot{B}_f \dot{U}_{f\ out} = \dot{B}_f K \dot{U}_{fgc}$ where $\dot{B}_f$ is the feedback coefficient.

$$\dot{U}_{\text{фск}} = \dot{E}_{\Phi} - \dot{\beta}_{\Phi} K \dot{U}_{\text{фск}},$$

fgc

From which it follows that

$$\dot{U}_{\Phi c\kappa} = \dot{E}_\Phi / (1 + K\dot{\beta}_\Phi).$$

The current of the equivalent generator of flicker-noise is equal to the voltage between the grid and the cathode multiplied by the total input admittance, or the mean square of the noise current is equal to:

$$\overline{I}^2_\Phi = |Y_{oc}|^2 \overline{U}^2_{\Phi c\kappa} = |\underset{\text{feedback}}{Y_{oc}}|^2 A f^{-\alpha} \Delta f / |1 + K\dot{\beta}_\Phi|^2.$$

Now to compute the noise voltage at the output of the amplifier we may use the equivalent scheme shown in figure 1 by introducing into it the current generators for the basic sources of noise as shown in figure 3.

The feedback coefficients β_f differ for various schemes and the expressions for the degree of feedback $1 + K\beta_f$ as shown in Table 2.

Table 2

I. Without feedback	II. With negative feedback without compensation	III. With compensation according to Pelchowitch	IV. With Compensated positive feedback
$\dot{\beta}_\varphi \approx 0$	$\dot{\beta}_\varphi = \frac{Y}{Y + Y_{\theta x}}$	$\dot{\beta}_\varphi = \dot{\beta}_\kappa \frac{Y}{Y + Y_{\theta x}}$	$\dot{\beta}_\varphi = \frac{Y}{Y + Y_{\theta x}} - \beta \frac{Y_+}{Y_+ + Y_{\theta x}}$
$1 + K\dot{\beta}_\varphi = 1 = \frac{Y_{oe}}{Y + Y_{\theta x}}$, where $Y_{oe} = Y + Y_{\theta x}$	$1 + \kappa\dot{\beta}_\varphi = \frac{Y(1+\kappa) + Y_{\theta x}}{Y + Y_{\theta x}} = \frac{Y_{oe}}{Y + Y_{\theta x}}$	$1 + \kappa\dot{\beta}_\varphi = \frac{Y(1 + \kappa\dot{\beta}_\kappa) + Y_{\theta x}}{Y + Y_{\theta x}} = \frac{Y_{oe}}{Y + Y_{\theta x}}$	$1 + \kappa\dot{\beta}_\varphi = \frac{YY_+[1+\kappa(1-\beta)] + Y_{\theta x} Y_{oe}}{(Y + Y_{\theta x})(Y_+ + Y_{\theta x})} \approx \frac{\frac{C_+}{C_{\theta x}} Y[1+\kappa(1-\beta] + Y_{oe}}{Y + Y_{\theta x}} \approx \frac{Y_{oe}}{Y + Y_{oe}}$ for $Y_{\theta x} \gg Y_+$ or $C_{\theta x} \gg C_+$
$\lvert Y_{oe}\rvert^2 = R^{-2} + \omega^2(C + C_{\theta x})^2 \approx R^{-2}(1 + \omega^2 R^2 C^2_{\theta x})$ for $C_{\theta x} \gg C$	$\lvert Y_{oe}\rvert^2 = \kappa^2 R^{-2} + \omega^2(\kappa C + C_{\theta x})^2 \approx \kappa^2 R^{-2}(1 + \omega^2 R^2 C^2)$ for $\kappa \gg 1$ u $\kappa C \gg C_{\theta x}$	$\lvert Y_{oe}\rvert^2 \approx R^{-2}(\kappa^2 + \omega^2 R^2 C^2_{\theta x})$ for $\kappa \gg 1$ u $\tau \approx \tau_\kappa$	$\lvert Y_{oe}\rvert^2 \approx R^{-2}(\kappa^2 + \omega^2 R^2 C^2_{oe})$ where $C_{oe} \approx C_{\theta x} + \kappa(C - \beta C_+)$ for $\kappa \gg 1$

As we can see from the Table in the first three schemes, the percentage of feedback through the admittances Y_{oc}, Y and Y_{in} is expressed in the same way. In the last scheme the expression for the percentage of coupling may be approximately represented in the same form if we use the normally encountered condition $Y_{in} \gg Y_+$ or $C_{in} \gg C_+$ (as a rule $C_+ < 0.05\ C_{in}$). Then the total noise voltage at the output of the electrometric amplifier (regardless of its schematic) is approximately equal to

noise
$$\overline{U}^2_{\text{ш вых}} = K^2\overline{U}^2_{\text{шск}} = K^2\overline{I}^2_{\text{ш}}/|Y_{\text{oc}}|^2 = (K^2/|Y_{\text{oc}}|^2)\,(\overline{I}^2_{\text{д}} + \overline{I}^2_R + \overline{I}^2_{\Phi})$$

or

$$\overline{U}^2_{\text{ш вых}} = (K^2/|Y_{\text{oo}}|^2)\,(2eI_0 + 4kTR^{-1} + Af^{-\alpha}\,|Y + Y_{\text{вх}}|^2)\,\Delta f.$$

If the signal current at the input of the amplifier is equal to I, then the square of the signal voltage at the output will be

signal
$$U^2_{\text{сигн. вых}} = K^2I^2/|Y_{\text{oc}}|^2$$

and the signal-to-noise ratio, which is usually determined as the ratio of the signal power to the noise power at the output, will be equal to

$$\text{с/ш} = U^2_{\text{сигн. вых}}/\overline{U}^2_{\text{ш вых}} = I^2/\overline{I}^2_{\text{ш}}.$$

The maximum (threshold) sensitivity is limited by the value of the total noise current at the input:

$$\overline{I}^2_{\text{ш}} = (2\,eI_c + 4\,kTR^{-1} + Af^{-\alpha}\,|Y + Y_{\text{вх}}|^2)\,\Delta f.$$

This current does not depend directly on the schematic of the amplifier or on the presence of feedback; however, because for a given band-pass the selection of the value of R depends substantially on the schematic (e.g., when compensation is introduced R may be increased), the noise current depends indirectly on the schematic and on the presence of feedback and compensation.

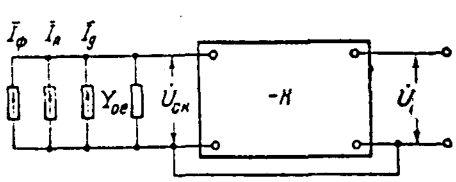


Figure 3. Generalized equivalent scheme of the amplifier with basic sources of internal noise. /97

2. The Variation in the Spectral Noise Density as a Function of the Parameters of the Circuit.

The spectral density of the total noise

$$G = d\bar{I}^2_{in}/df = 2eI_c + 4kTR^{-1} + Af^{-\alpha}|Y + Y_{Bx}|^2$$

depends on the parameters R, C, C_{in} and K, on the parameters of the electrometric tube I_c, A and α and on the frequency (due to flicker-noise). To compute the flicker-noise it is necessary to know the values of coefficients A and α. These coefficients were measured for several types of domestic electrometer tubes and core tubes which are suitable for operation in the electrometric state (Table 3). On the basis of these measurements we may assume that for modern electrometric tubes of the subminiature series the coefficient of flicker-noise A = (2-0.2) 10^{11} volt^2 $\text{cycles}^{1-\alpha}$ with the exponent α having a value between 1.6 and 1.

Table 3. Average results of measurements involving not less than three tubes of each type.

Type of tube	2E2P	I-1	FM-7	1zh42A
$A.10^{11}$ volt^2 eff $\text{cycles}^{1-\alpha}$	2.5	2	0.2	0.2
α	1.6	1.6	1	1.4

Operating conditions: the heaters operated as specified in the handbook, U_α = 6-108, $I_c \leq 10^{-14}$

The parameters of the input circuit of electrometric amplifiers usually lie within the following limits: input capacity $C_{in} \geq 5$ pf; the capacity of the feedback loop $C \geq 0.2$ pf; the gain without feedback $K \geq 10^3$. The resistance of the feedback loop $R = 10^{12} - 10^{11}$ ohm. In order to compare the scheme with feedback with a scheme without feedback when the current sensitivity is the same $\rho = U_{out} I^{-1} = |Y_{oc}|^{-1}$, in the scheme without feedback the resistance R must be selected so that it is less by a factor of 1 + K. For example if in the scheme with feedback $R = 10^{12}$ ohm, $K = 10^3$ and $\rho = KR/(1 + K) = 10^{12}$ ohm, then in the scheme without feedback we obtain the same value $\rho = KR$ when $R = 10^9$ ohm. The grid current of the electrometric tube usually does not exceed 10^{-14} amp.

To evaluate the relative magnitude of the noise components and to clarify the variation in the total noise as a function of frequency we assume the following values for the parameters: $C_{in} = 5$ pf, $C = 0.2$ pf, $K = 10^3$, $I_c = 10^{14}$a. Figure 4 shows that spectral density of the noise current components as a function of the frequency for $A = 2 \times 10^{-11}$ v^2 $cyc^{0.6}$ and $\alpha = 1.6$, and various values of R from 10^{12} to 10^9 ohm; figure 5 also shows curves for tubes of other types which have $A = 0.2 \times 10^{-11}$ volts^2 and $\alpha = 1$. It follows from these curves that in the schemes considered the spectral density of the flicker-noise has a minimum at frequency of f^*, which is determined from the condition /98

$$\frac{d}{df}\left(\frac{d\bar{I}_\Phi^2}{df}\right) = \frac{d}{df}[Af^{-\alpha}R^{-2}(1 + 4\pi^2R^2C_{\text{вх}}^2f^2)],$$

from which

$$f^* = (2\pi RC_{\text{вх}})^{-1}\sqrt{\alpha/(2-\alpha)}.$$

The greater the time constant for the input circuit without feedback $\tau_{in} = RC_{in}$, the lower the frequency of the f^* minimum. At frequencies substantially below f^*, spectral density of the flicker-noise increases according to the law $AR^{-2} f^{-\alpha}$; at frequencies substantially greater than f^* it increases in accordance with the law $AC_{in}^2 f^{2-\alpha}$. The greater f^*, the slower is the increase in the spectral density as the frequency increases. The spectral

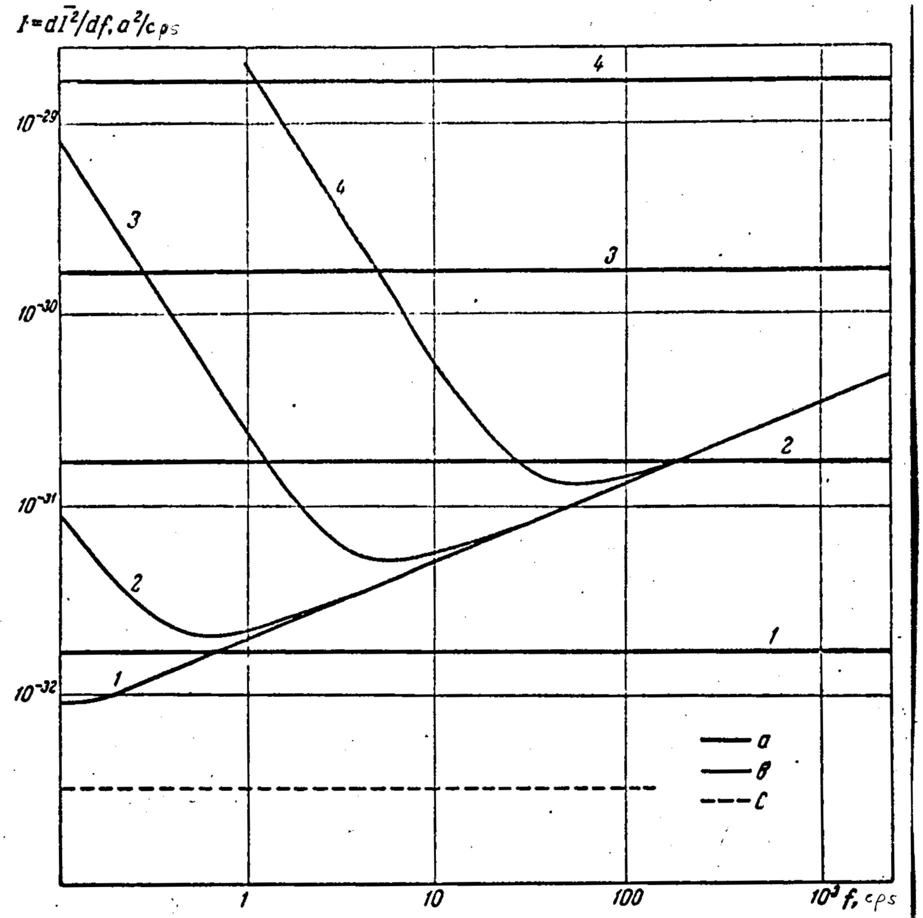


Figure 4. The spectral density of noise current components as a function of frequency for tube 1 Zh42A for various values A, α and $R.A = 0.2 \times 10^{-11} v^2 \, cyc^{0.6}$, $\alpha = 1.6$. a--thermal noise, b--flicker-noise, c--shot noise of the grid current when $I_c = 10^{-14}$a.

R	10^{12}	10^{11}	10^{10}	10^{9}
N	1	2	3	4

density of thermal noise, naturally, does not depend on the frequency; however, as R increases it becomes smaller and the minimum value of the spectral density /99 of the flicker-noise also becomes smaller. The spectral density of shot noise produced by the current for $I_c = 10^{-14}$ is lower by almost 1 order than the spectral density of the thermal noise for the maximum considered value of $R = 10^{12}$ ohm; therefore, in the future we may neglect it.

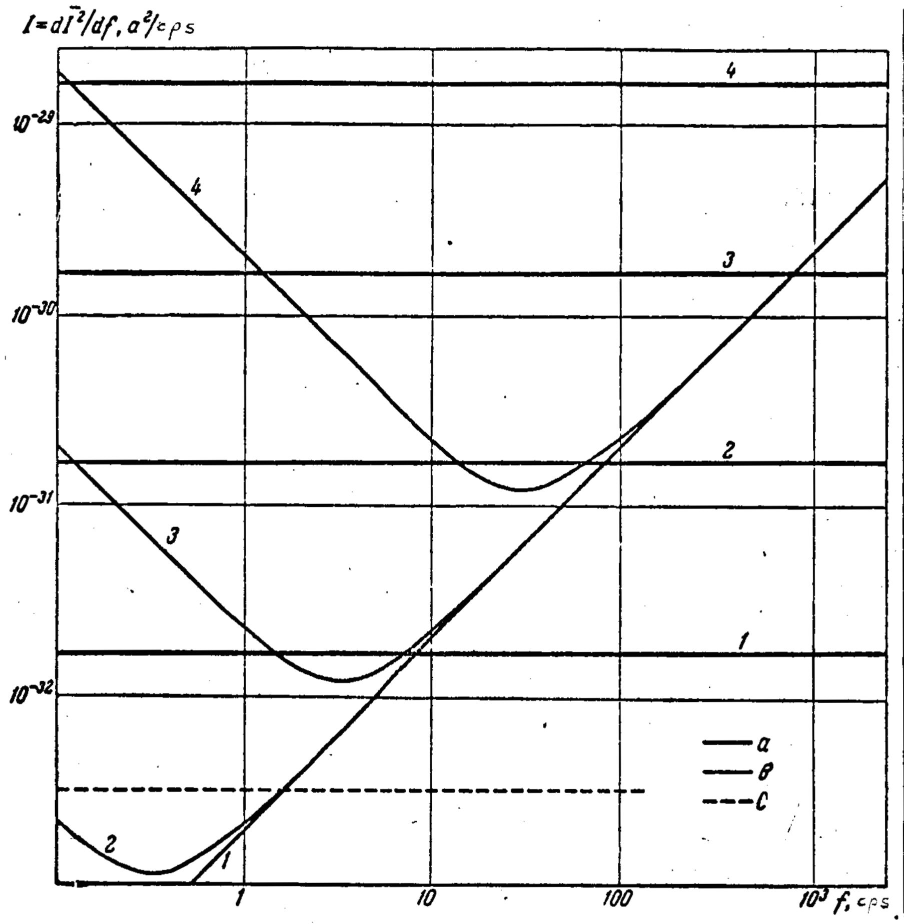


Figure 5. The spectral density of the noise current components for the tube EM-7. $A = 0.2 \times 10^{-11}$ v^2 cyc^0, $\alpha = 1$. a--thermal noise, b--flicker-noise, c--shot noise of the grid current when $I_c = 10^{-14}$ amp.

R	10^{12}	10^{11}	10^{10}	10^9
N	1	2	3	4

For all values of R there is a certain band of frequencies within which the spectral density of the flicker-noise is less than that of the thermal noise. Figure 6 shows the curves for the total noise current per unit band-width as a function of frequency for the same values of R,A and α which were used to construct the curves of figures 4 and 5. The solid lines show the

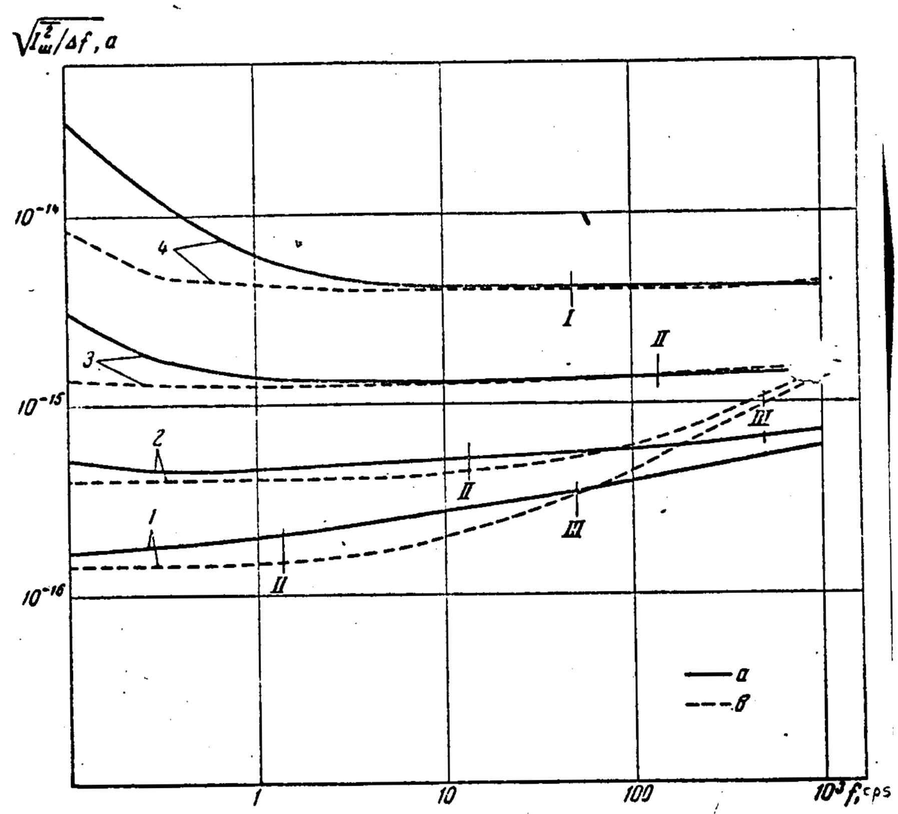


Figure 6. The total noise current per unit band-width for the tube 1Zh42A (solid line) and EM-7 (broken line). a--A = 2 x 10^{-11} v^2 $cyc^{-0.6}$, α = 1.6; b--A = 0.2 x 10^{-11} $v^2 cyc^0$, α=1.

R	10^{12}	10^{11}	10^{11}	10^9
N	1	2	3	4

curves for the tubes with a strong dependence of flicker-noise on the frequency (α= 1.6); the broken lines show the curves for tubes with a weaker dependence of flicker-noise on frequency ($\alpha = 1$). In the case when $\alpha = 1.6$ for $R \leq 10^{11}$ ohm, the noise current in the range of frequencies from 1 cps to 1 kc is practically independent of the frequency, i.e., we have a predominance of thermal noise. At the higher frequencies the noise current will increase since the spectral density of the flicker-noise will become greater than the spectral sensity of the thermal noise; however, this need not lead to a noticeable increase in the noise voltage at the output because the frequency characteristics of current sensitivity (figure 7) (even when compensation is present) decrease at frequencies above 1 kc. With a resistance of 10^{12} ohm in the pass band, /100 which is limited in the circuit with compensation to a frequency of 50 cps, the thermal noise also is dominant. The boundary frequencies f_B corresponding to the decrease in the current sensitivity to $0.7\,\rho_{max}$ are marked on figure 6 and 7 by vertical lines. In the case $\alpha = 1$ the curves for the total noise current are steeper; however, in this case too, only in the schemes having compensation do we observe an insignificant domination of the flicker-noise. On the other hand in the region of lower frequencies the noise increases much slower with these tubes.

Until now we have considered the spectral density of noise or of the noise current in a narrow band of frequencies. To compute the total noise it is necessary to indicate the spectral density of the noise voltage at the output over the entire pass band of the amplifier;

$$\overline{U}^2_{\text{noise out}} = K^2R^2 \int_0^{f_B} (1 + 4\pi^2\tau^2_{oc}f^2)^{-1} \; 2eI_c +$$

$$+ 4k\,TR^{-1} + AR^{-2}f^{-\alpha}(1 + 4\pi^2\tau^2_{in}\; f^2)\; df.$$

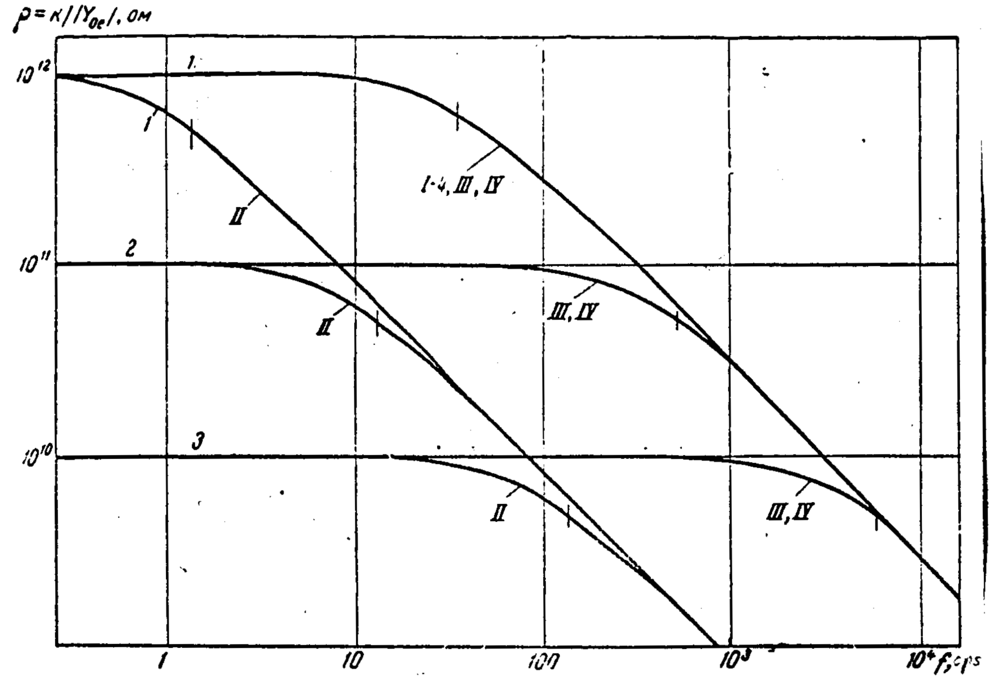


Figure 7. The sensitivity to current $\rho = \left|\frac{U_{\text{вых}}}{I}\right| = \frac{K}{|Y_{\text{oc}}|}$ as a function of frequency. I--without feedback; II--with feedback without compensation; III, IV--with compensation.

R	10^{12}	10^{11}	10^{11}	10^{9}
N	1	2	3	4

In this connection difficulties arise associated with the fact that in the limits from zero to the upper boundary frequency f_B the integral diverges due to the term $AR^{-2} f^{-\alpha}$ when $\alpha \geq 1$. The lower limit $f_e = 0$ takes into account the noise components (and signal) with an infinitely large period, which corresponds to an infinitely large period of measurement. In practice the measurement time t is always finite and the lower limit of integration may be limited by the frequency $f_e = t^{-1}$.

A large measurement time is not required in wide band electrometric amplifiers. Usually $t = 10 - 100$ sec and $f_e = 0.1$ to 0.01 cps. The curves on figure /101
6 cover a range of frequencies which are typical for such amplifiers. Almost in the entire range the thermal noise predominates; the spectral density of the total noise depends little on the frequency and the mean square of the noise voltage at the output is practically porportional to the band-width of the amplifier.

Conclusions

In wide band amplifiers with compensated frequency characteristics when we use modern electrometric tubes of the subminiature series, the thermal noise of the resistance predominates and consequently the sensitivity is close to the theoretical limit. In amplifiers which have high sensitivity to current and whose pass band is reduced as much as possible ($f_B = 1 - 5$ cps) to decrease the noise voltage, the flicker-noise predominates. In such amplifiers it is desirable to use tubes with a small value of α.

In most cases to increase the signal-to-noise ratio it is advantageous to increase R and (when necessary) increase the pass band by means of compensation. The better the tube (from the point of view of flicker-noise), i.e., the lower the flicker-noise constant A, the greater may be the value of R. The value of the input capacity C_{in} must be as small as possible since an increase in Y produces an increase in the spectral density of the flicker-noise current at the input.

The variation in the signal-to-noise is a function of the circuit scheme which is only manifested by the fact that in a scheme with feedback and with compensation it permits an increase in the value of resistance R for a given band-width and thereby produces a decrease in the noise currents at the input.

As far as the method of compensation is concerned, as we see from reference 5 both methods (positive feedback and compensating filter in the negative feedback loop) produce approximately the same increase in the band-width.

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